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Final Exam, MTH 213, Spring 2019

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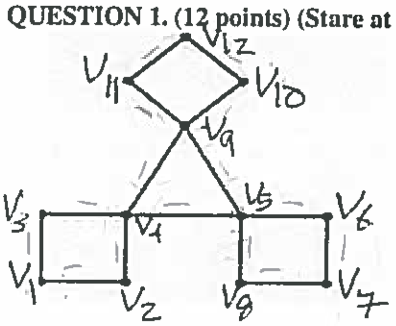
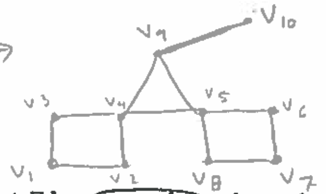
SCORE = $\frac{70}{78}$ (☐ (*MW*) ☑ (*UTR*)

QUESTION 1. (12 points) (Stare at the following graph, say *G*).



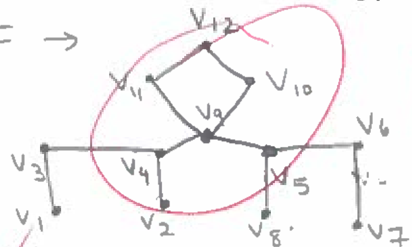
(i) Let *F* be an induced subgraph of G such that *F* has exactly 10 vertices. DRAW *F*.

F →



(ii) Let *F* be a spanning subgraph of G such that *F* is a tree. Draw *F*.

F →



NO NO

(iii) Is *G* a bipartite? explain

no, G is not bipartite because it has an odd cycle $V_9 \overset{1}{-} V_5 \overset{2}{-} V_4 \overset{3}{-} V_9$

(iv) Is *G* an Euler Circuit (Eulerian)? explain.

yes, G is a Euler Circuit because every edge is of even degree, and we can have a circuit that visits every edge once and returns to the first vertex:
$V_{12} - V_{11} - V_9 - V_4 - V_3 - V_1 - V_2 - V_4 - V_5 - V_8 - V_7 - V_6 - V_5 - V_9 - V_{10} - V_{12}$

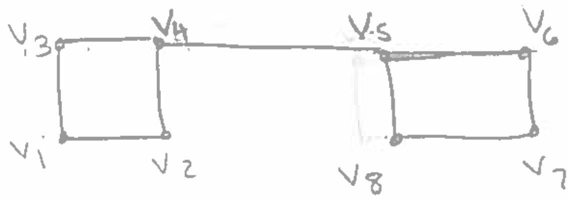
(v) Is *G* Hamiltonian? explain

no, G is not hamiltonian, because there is no cycle possible that visits every vertex only once and returns to the first vertex

(vi) Let *F* be an induced subgraph of *G*. Suppose that *F* is an Euler trail (path) that is not an Euler Circuit and *F* has more than 6 vertices. Draw *F*

↳ exactly 2 vertices of odd degree

F →



*V_4 and V_5 are of degree 3

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QUESTION 2. (4 points) (SHOW STEPS) Let $d = gcd(124, 348)$. Find d, then find a, b such that $d = 124a + 348b$.

$348 = 124(2) + 100$

$124 = 100(1) + 24$

$100 = 24(4) + 4$

$24 = 4(6) + 0$

$100 - 24(4) = 4$

$100 - [124 - 100](4) = 4$

$100 - 124(4) + 100(4) = 4$

$100(5) - 124(4) = 4$

$(348 - 124(2))5 - 124(4) = 4$

$348(5) - 124(2)(5) - 124(4) = 4$

$\rightarrow 348(5) - 124(14) = 4$

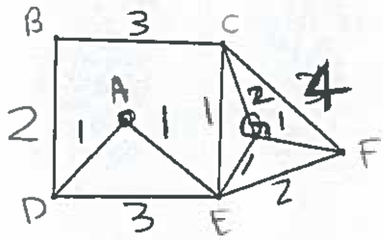
$d = 4$

$a = -14$

$b = 5$

$(4) = 124(-14) + 348(5$

QUESTION 3. (5 points) Stare at the following graph



A Post office is located at vertex A. The MAIL man needs to visit each block (vertex) exactly once and then come back to vertex A. Find the minimum weight-cycle that he should use.

$\rightarrow A \xrightarrow{1} D \xrightarrow{2} B \xrightarrow{3} C \xrightarrow{2} G \xrightarrow{1} F \xrightarrow{2} E \xrightarrow{1} A$

$\rightarrow$ weight $= 12$

Try more!

QUESTION 4. (6 points) (SHOW STEPS) Let X be the temperature of some village in Russia. Given $-90 < X < 0$, $X \equiv 4\ (mod\ 9)$ and $X \equiv 8\ (mod\ 10)$. Find the value of X.

CRT: * $\gcd(9,10)=1 \rightarrow n = 9(10) = 90$

$m_1 = \frac{90}{9} = 10 \rightarrow m_1^{-1} \text{ in } Z_9 = 1$

$m_2 = \frac{90}{10} = 9 \rightarrow m_2^{-1} \text{ in } Z_{10} = 9$

$X = (4(10)(1) + 8(9)(9)) \bmod 90$

$X = (688) \bmod 90 = 58$

$x = 58 + 90K$

X b/w $-90 < x < 0$:

$k = -1$

$x = 58 + 90(-1)$

$x = -32$

QUESTION 5. (3 points) There are 400 balls. Each ball is either RED or BLUE OR GREEN OR BLACK. n balls are selected randomly. What is the minimum value of n that will secure at least 70 balls are of the same color. SHOW the work

D

$D \rightarrow 400$

$C \rightarrow 4$

$\hookrightarrow \left\lceil \frac{n}{4} \right\rceil = 70$

$n = 70(4) = 280$

QUESTION 6. (Show steps)

(i) **(3 points)** What is $5^{1203}\ (mod\ 21)$?

$m = 21 = 3 \cdot 7$

$\phi(21) = (3-1)(7-1)$
$= 12$

$\underbrace{5^{12 \times 1000}} \cdot 5^3 (\bmod 21)$

$1 \cdot 5^3 (\bmod 21) = 20$

(ii) **(3 points)** Find all possible values of X over Planet Z_{14} where $4X = 6$.

$\gcd(4,14) = 2$

$2 | 6 \rightarrow$ yes, $\underline{2}$

possible values

$X = 5$

$\xrightarrow{+7}$

$X = 12$

(iii) **(2 points)** Find all possible values of X over Planet Z where $4X \equiv 6\ (mod\ \cancel{21}\ 14)$.

$X = 5 + 7K$

(iv) **(3 points)** Let $n = (49)(125) = 6125$. How many positive integers < 6125 such that $gcd(each\ integer, 6125) = 7$.

$\frac{n}{d} = \frac{6125}{7} = \boxed{875} = 5 \cdot 5 \cdot 5 \cdot 7 = 5^3 \cdot 7$

$\phi(\frac{n}{d}) = (5-1)5^2(7-1) = \boxed{600}$

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QUESTION 7. (6 points)) Consider the following code

For $k = 2$ to (n^2+1)
$S = k^4 + 3*k^2 + K + 4$ → 8
For $i = 1$ *to* k
$L = k^2 + 7*k + 3$ → 4
Next i
For $m = 2$ to $(3k+1)$
$H = 7*m^3 + 6*m^2 + 10$ → 7
next m
next k

(i) Find the exact number of addition, subtraction, multiplication that the code executed.

Outer loop:
* will run: $(n^2+1) - 2 + 1 = n^2$
* # of operations: $8(n^2)$

1st inner loop:
* will run: $K - 1 + 1 = K$
* # of operations when $K=2$: $4(2) = 8$
* # of operations when $K = n^2+1$: $4(n^2+1)$

2nd inner loop:
* will run: $3K + 1 - 2 + 1 = 3K$
* # of operations when $K = 2$: $7(3(2)) = 42$
* # of operations when $K = n^2+1$: $7(3(n^2+1)) = 21(n^2+1)$

# of operations the code will run $= 8n^2 + \left[\frac{(8 + 4(n^2+1))(n^2)}{2}\right] + \left[\frac{(42 + 21(n^2+1))(n^2)}{2}\right]$

(ii) Find the complexity of the code. $O(\text{code}) = n^4$

QUESTION 8. (4 points) There are 204 nonzero negative integers and 500 nonzero positive integers. Then there are at least n integers, say $a_1, a_2, ..., a_n$ such that $a_1 \ (mod\ 7) = a_2 \ (mod\ 7) = ... = a_n \ (mod\ 7)$. Find the maximum value of n. SHOW THE WORK

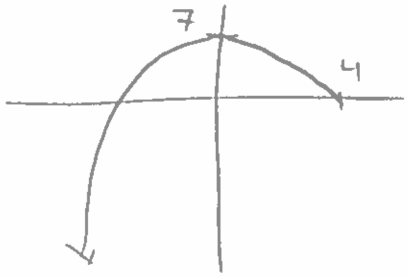
204 −ve and 500 +ve → no zeros

D → total integers = 704

C → 7 possibilities

$n = \left\lceil \frac{704}{7} \right\rceil = \lceil 100.57 \rceil = 101$

QUESTION 9. (3 points) Let $f : (-\infty, 4] \to (-\infty, 7]$ be a function that is ONTO such that $f(x) = b - x^2$. Find the value of b



since the function is onto range = codomain

So $b = 7$

QUESTION 10. (3 points) Let $f : (-3.5, 9) \to (1, 6)$ such that $f(x) = \sqrt{2x+7} + 1$. Then f is bijective, find $f^{-1}(x)$. Then find the domain and the codomain of f^{-1}.

$X = \sqrt{2y+7} + 1$

$(X-1)^2 = 2y+7$

$f^{-1}(x) \to \boxed{y = \frac{(x-1)^2 - 7}{2}}$

domain codomain

$f^{-1} : (1, 6) \longrightarrow (-3.5, 9)$

QUESTION 11. (3 points) Let $A = \{0, 3, 4, 5, 7, 8\}$ Define "$\leq$" on A such that $a \leq b$ if and only if $(a-b) \pmod 4 \in \{0, 2\}$. Convince me that "$\leq$" is not a partial order relation on A.

$\to (5-3) (\text{mod} 4) = 2 \therefore 5 " \leq " 3$

$\to (3-5) (\text{mod} 4) = 4 - 2 (\text{mod} 4) = 2 \therefore 3 " \leq " 5$

$\to$ "$\leq$" is not a partial order because it is not antireflexive (2nd rule Fails)

QUESTION 12. (6 points) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 21, 35, 40\}$. Define "=" on A such that "$a = b$" if and only if $(a-b) \ (mod\ 12) \in \{0, 4, 8\}$. Then "=" is an equivalence relation.

1) Find all equivalence classes of A.

$[1] = \{1, 5, 9, 21\}$ ← 4 elements

$[2] = \{2, 6, 10, 14\}$ ← 4 elements

$[3] = \{3, 7, 11, 35\}$ ← 4 elements

$[4] = \{4, 8, 40\}$ ← 3 elements

2) If we view "=" as a subset of $A \times A$, how many elements does "=" have?

$|"="| = 4 \times 4 + 4 \times 4 + 4 \times 4 + 3 \times 3$

$= 57$

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QUESTION 13. (4 points) Use the four-method and convince me that $\sqrt{29}$ is irrational number.

$\rightarrow \sqrt{29}$ is a rational number $\sqrt{29} = \frac{a}{b}$ + a, b are odd integers

$29 = \frac{a^2}{b^2} \leftarrow \frac{(2n+1)^2 \text{ - for some } n \in Z \text{ (odd)}}{(2m+1)^2 \text{ - for some } m \in Z \text{ (odd)}} \rightarrow 29 = \frac{4n^2+4n+1}{4m^2+4m+1}$

$\frac{29(4)m^2 + 29(4)m + 28}{4} = \frac{4n^2+4n}{4} \rightarrow 29m^2 + 29m + 7 = n + n$

odd odd odd odd +

odd $\neq$ even

contradiction inval

$\sqrt{29}$ is irrationa

X 0 2

QUESTION 14. (8 points) Use math-induction and convince me that $7 \mid (2^{(6n+2)} + 6^{(2n)} - 5)$ for every $n \geq 1$.

① for n=1: $7 \mid (2^{6(1)+2} + 6^{2(1)} - 5)$

$\hookrightarrow \frac{2^8 + 6^2 - 5}{7} = 41$ integer $\rightarrow$ true

② we assume $7 \mid (2^{6n+2} + 6^{2n} - 5)$ for some integer n

③ for n+1: $7 \mid (2^{6(n+1)+2} + 6^{2(n+1)} - 5)$

$7 \mid (2^{6n} \cdot 2^8 + 6^{2n} \cdot 6^2 - 5) + 2^8 - 2^8 + 6^2 + 6$

$7 \mid 2^8(2^{6n}+1) + 6^2(6^{2n}+1) - 5 - 2^8 - 6^2$

$7 \mid \underbrace{(2^8 + 6^2 - 5)}_{\text{true by ①}} \underbrace{(2^{6n} + 6^{2n} - 5)}_{\text{true by ②}}$

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Faculty information

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